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21

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A Commutation Formula in Continuum Mechanics

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CONTENTS

INTRODUCTION

In a previous work [1] I have assumed the existence of a general type of convection-diffusion theorem in continuum mechanics, and have studied the properties of this type of theorem. In this report I demonstrate that, given a tensor of any order associated with the motion of a continuum, at least one such theorem exists.

0. MATHEMATICAL PRELIMINARIES

I use, essentially, the notations and assumptions of Truesdell and Toupin [2]. A short synopsis is included here in order to make this work essentially self-contained. It is emphasized that this work is formal in nature. In particular, statements concerning smoothness conditions on functions are seldom made. In general it is understood, as in formal works on differential geometry, that all functions are smooth enough to accommodate the manipulation performed and render the results meaningful. No distinction is made between a function and its values, except where confusion would result if this distinction were not made.

The results derived here are valid in Euclidean three-dimensional space. In such a space a rectangular Cartesian coordinate system always exists. We denote points in this system by the symbols $\underset{\sim}{Z}, \underset{\sim}{z}$. A <u>motion</u> is a one-parameter mapping

$$\underset{\sim}{z} = \hat{\underset{\sim}{z}}(\underset{\sim}{Z}, t), \quad \underset{\sim}{Z} = \hat{\underset{\sim}{Z}}(\underset{\sim}{z}, t), \tag{0.1}$$

where t is a real-valued parameter interpreted as the time. A point $\underset{\sim}{Z}$ is called a <u>particle</u> and a point $\underset{\sim}{z}$ is called a <u>place</u>, or space-point.

We choose a single curvilinear coordinate system given by

$$\underset{\sim}{x} = \underset{\sim}{g}(\underset{\sim}{z}), \quad \underset{\sim}{X} = \underset{\sim}{g}(\underset{\sim}{Z}), \tag{0.2}$$

where $\underset{\sim}{g}$ is the same function in both equations. The $\underset{\sim}{X}$ are called material coordinates, and the $\underset{\sim}{x}$ are called spatial coordinates.

It is convenient to use two systems of notation. The notation of general, double tensors is used when it is desired to emphasize the components of an equation*. Here capital Roman** indices denote quantities which transform as tensors with respect to changes in the material coordinates $\underset{\sim}{X}$, and lower-case Roman indices denote quantities which transform as tensors with respect to the spatial coordinates $\underset{\sim}{x}$***. The diagonal summation convention is used, and all indices have the values 1,2,3.

When display of indices would cloud the physical significance of an equation, direct notation is used.

Let arc lengths at $\underset{\sim}{X}$ and $\underset{\sim}{x}$ be given by

$$
\begin{aligned}
dS^2 &= g_{AB}dX^A dX^B , \\
ds^2 &= g_{ab}dx^a dx^b ;
\end{aligned}
\qquad (0.3)
$$

g_{AB} and g_{ab} are the components of the contravariant material and spatial metric tensors. Cristoffel symbols based on these tensors are defined in the usual manner.

Let $\varphi^{\cdots}_{\cdots}(\underset{\sim}{x},\underset{\sim}{X})$ be the components of a mixed double tensor field. The notation $\varphi^{\cdots}_{\cdots ; A}$ denotes the covariant derivative of ϕ

*Double tensor fields are discussed in greater detail in [3].

**This is a slight departure from the notation of [1], which uses lower-case Greek indices in this situation and upper-case Roman indices when the spatial and material coordinates are independently selected.

***The exceptions are, of course, the sets of coordinates $\underset{\sim}{X}$, X^A; $\underset{\sim}{x}$, x^a.

with respect to X^A when the x^a are held constant. This derivative is called the partial covariant derivative*, and is denoted by a comma.

$$\phi^a_{A,B} = \frac{\partial \phi^a_A}{\partial X^B} - \left\{ {C \atop A\ B} \right\} \phi^a_C \,, \tag{0.4}$$

where $\partial/\partial X^B$ is performed with all of the x^a, and all of the X^D, $D \neq B$, held constant.

The partial covariant derivative with respect to spatial coordinates is defined in an analogous manner.

Let a one-parameter mapping between $\underset{\sim}{x}$ and $\underset{\sim}{X}$ be given by the composite of (0.1) and (0.2),

$$\underset{\sim}{x} = \hat{\underset{\sim}{x}}(\underset{\sim}{X},t) \,, \quad \underset{\sim}{X} = \hat{\underset{\sim}{X}}(\underset{\sim}{x},t) \,. \tag{0.5}$$

The total covariant derivative of a mixed double field $\phi^{\cdots}_{\cdots}(\underset{\sim}{x},\underset{\sim}{X})$ is given by

$$\phi^{\cdots}_{\cdots;A} = \phi^{\cdots}_{\cdots,A} + \phi^{\cdots}_{\cdots,a}\frac{\partial x^a}{\partial X^A} \,. \tag{0.6}$$

This derivative has the properties that when $\phi^{\cdots}_{\cdots}$ is of the form $\phi^{A..B}_{C..D}(\underset{\sim}{X})$ or $\phi^{a..b}_{c..d}(\underset{\sim}{x})$, then $\phi^{\cdots}_{\cdots;E}$ reduces to $\phi^{\cdots}_{\cdots,E}$ or $\phi^{\cdots}_{\cdots,e}\dfrac{\partial x^e}{\partial X^E}$ respectively, and if the operation $;E$ is performed on $\phi(\underset{\sim}{x},\underset{\sim}{X})$ then

* This discussion on partial and total covariant derivatives is based on [3].

$\underset{\sim}{x}$ is replaced by $\hat{\underset{\sim}{x}}(\underset{\sim}{X},t)$, the same result is obtained as if the operation ;E were performed on $\phi{:::}(\hat{\underset{\sim}{x}}(\underset{\sim}{X},t)\underset{\sim}{X})$.

Let

$$\phi^{\cdots}_{\cdots;a} = \phi^{\cdots}_{\cdots,a} + \phi^{\cdots}_{\cdots,A} \frac{\partial X^A}{\partial x^a} . \tag{0.7}$$

If we define the Kronecker delta in the usual manner, then

$$\frac{\partial x^a}{\partial X^A} \frac{\partial X^A}{\partial x^b} = \delta^a_b , \quad \frac{\partial X^A}{\partial x^a} \frac{\partial x^a}{\partial X^B} = \delta^A_B , \tag{0.8}$$

and by (0.6) and (0.7)

$$\phi^{\cdots}_{\cdots;A} = \phi^{\cdots}_{\cdots;a} \frac{\partial x^a}{\partial X^A} , \tag{0.9}$$

$$\phi^{\cdots}_{\cdots;a} = \phi^{\cdots}_{\cdots;A} \frac{\partial X^A}{\partial x^a} . \tag{0.10}$$

<u>Deformation</u> <u>gradients</u> are defined by

$$F^A_a = \frac{\partial X^A}{\partial x^a} = X^A_{\ ,a} , \qquad F^a_A = \frac{\partial x^a}{\partial X^A} = x^a_{\ ,A}, \tag{0.11}$$

and <u>composite</u> <u>deformation</u> <u>gradients</u> are given by

$$f^{A\ldots B}_{a\ldots b} = f^A_a \cdots f^B_b , \tag{0.12}$$

$$F^{a\ldots b}_{A\ldots B} = F^a_A \cdots F^b_B . \tag{0.13}$$

The <u>expansion</u> is the absolute scalar J given by

$$J = \frac{\sqrt{\det g_{ab}}}{\sqrt{\det g_{AB}}} \det \left(\frac{\partial x^a}{\partial X^A}\right) . \tag{0.14}$$

The <u>velocity</u> of a particle is given by

$$\dot{x}^a = \frac{\partial x^a}{\partial t} \tag{0.15}$$

where the partial derivative is taken with $\underset{\sim}{X}$ held constant.

Let $\phi(\underset{\sim}{x},\underset{\sim}{X},t)$ be a double tensor. The <u>material derivative</u> of ϕ is the double tensor with components

$$\frac{d}{dt}\phi^{\cdots}_{\cdots} = \frac{\partial \phi^{\cdots}_{\cdots}}{\partial t} + \phi^{\cdots}_{\cdots,g}\,\dot{x}^g . \tag{0.16}$$

where the partial time derivative is taken with both $\underset{\sim}{x}$ and $\underset{\sim}{X}$ held fixed, and $_{,g}$ denotes the partial covariant derivative with respect to x^g. This derivative is a double tensor whose value is independent of whether $\underset{\sim}{x}$ is replaced by $\hat{\underset{\sim}{x}}(\underset{\sim}{X},t)$. It is often convenient to write $\frac{d}{dt}(\ \ ) = (\dot{\ })$.

In this work, we adhere to the <u>convention</u> that <u>either the material or the spatial description is used</u>, <u>but not both</u>. That is, all functions are written in terms of $\underset{\sim}{x}$ and t, or in terms of $\underset{\sim}{X}$ and t.

It is well-known that*

$$\frac{d}{dt}(x^a{}_{;A}) = \dot{x}^a{}_{,A} = \dot{x}^a{}_{,b}\,x^b{}_{,A} . \tag{0.17}$$

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* [1], §76.

Let J be given by (0.14). Then Euler's expansion formula is

$$\dot{J} = J\,\dot{x}^k{}_{,k} \; . \tag{0.18}$$

The acceleration is defined by

$$\ddot{x}^a = \frac{d}{dt}(\dot{x}^a) \; . \tag{0.19}$$

1. A COMMUTATION FORMULA

Consider a double tensor field with components $\phi^{A\ldots Ba}_{C\ldots Dc}$. We have

$$\phi^{A\ldots Ba}_{C\ldots Dc,e} = \frac{\partial \phi^{A\ldots Ba}_{C\ldots Dc}}{\partial x^e} + \left\{ {a \atop f\ e} \right\} \phi^{A\ldots Bf}_{C\ldots Dc} - \left\{ {g \atop c\ e} \right\} \phi^{A\ldots Ba}_{C\ldots Dg}\,, \tag{1.1}$$

so that by (0.6)

$$\phi^{A\ldots Ba}_{C\ldots Dc,e}\, x^e{}_{,E} = \frac{\partial \phi^{A\ldots Ba}_{C\ldots Dc}}{\partial x^e} \frac{\partial x^e}{\partial X^E} + \left\{ {a \atop f\ e} \right\} \phi^{A\ldots Bf}_{C\ldots Dc} \frac{\partial x^e}{\partial X^E}$$

$$- \left\{ {g \atop c\ e} \right\} \phi^{A\ldots Ba}_{C\ldots Dg} \frac{\partial x^e}{\partial X^E}\,,$$

$$= \phi^{A\ldots Ba}_{C\ldots Dc;e}\, x^e{}_{,E}\,. \tag{1.2}$$

By (0.9), (1.2) becomes

$$\phi^{A\ldots Ba}_{C\ldots Dc,e}\, x^e{}_{,E} = \phi^{A\ldots Ba}_{C\ldots Dc;E}, \tag{1.3}$$

and by the convention that only the material or the spatial description is used, but not both, we have

$$\phi^{A\ldots Ba}_{C\ldots Dc,e}\, x^e{}_{,E} = \phi^{A\ldots Ba}_{C\ldots Dc,E}. \tag{1.4}$$

By (1.4) and the rule for differentiating products of functions

$$\frac{d}{dt}(\phi^{A\ldots Ba}_{C\ldots Dc,E}) = \frac{d}{dt}(\phi^{A\ldots Ba}_{C\ldots Dc,e}\, x^{e}{}_{,E})$$

$$= \frac{d}{dt}(\phi^{A\ldots Ba}_{C\ldots Dc,e})\, x^{e}{}_{,E} + \phi^{A\ldots Ba}_{C\ldots Dc,e}\, \frac{d}{dt}(x^{e}{}_{,E}) \,. \qquad (1.5)$$

By a commutation formula*, (1.5) becomes

$$\frac{d}{dt}(\phi^{A\ldots Ba}_{C\ldots Dc,E}) = \dot\phi^{A\ldots Ba}_{C\ldots Dc,e}\, x^{e}{}_{,E} - \phi^{A\ldots Ba}_{C\ldots Dc,k}\, \dot x^{k}{}_{,e}\, x^{e}{}_{,E}$$

$$+ \phi^{A\ldots Ba}_{C\ldots Dc,e}\, \frac{d}{dt}(x^{e}{}_{,E}) \,. \qquad (1.6)$$

By (0.17), (1.6) becomes

$$\frac{d}{dt}(\phi^{A\ldots Ba}_{C\ldots Dc,E}) = \dot\phi^{A\ldots Ba}_{C\ldots Dc,e}\, x^{e}{}_{,E} - \phi^{A\ldots Ba}_{C\ldots Dc,k}\, \dot x^{k}{}_{,e}\, x^{e}{}_{,E}$$

$$+ \phi^{A\ldots Ba}_{C\ldots Dc,e}\, \dot x^{e}{}_{,k}\, x^{k}{}_{,E} \,. \qquad (1.7)$$

The last two terms in (1.7) cancel, giving

$$\frac{d}{dt}(\phi^{A\ldots Ba}_{C\ldots Dc,E}) = \dot\phi^{A\ldots Ba}_{C\ldots Dc,e}\, x^{e}{}_{,E} \,. \qquad (1.8)$$

By (1.4), (1.8) is

$$\frac{d}{dt}(\phi^{A\ldots Ba}_{C\ldots Dc,E}) = \dot\phi^{A\ldots Ba}_{C\ldots Dc,F} \,; \qquad (1.9)$$

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* [2], p. 338.

for a tensor of the form considered $\frac{d}{dt}$ and the partial covariant derivative "$,_E$" commute.

In fact, the proof given above extends easily to the general case of any tensor function with any combination of covariant and contravariant, material and spatial indices. That is, for any tensor function ϕ of the type considered here, "$\frac{d}{dt}$" and "$,_E$" commute.

$$\left(\frac{d\phi}{dt}\right)_{,E} = \frac{d}{dt}(\phi_{,E}) ,$$

$$\text{GRAD } \dot{\phi} = \dot{\overline{\text{GRAD } \phi}} . \tag{1.10}$$

A formula analogous to (1.10) but containing a spatial gradient is familiar. Using (1.10) and an obvious generalization of (1.4), we have

$$\left(\frac{d\phi}{dt}\right)_{,e} x^e_{,E} = \frac{d}{dt}(\phi_{,e}\, x^e_{,E}) . \tag{1.11}$$

By the lemma (0.17), this becomes

$$\left(\frac{d\phi}{dt}\right)_{,e} x^e_{,E} = \frac{d}{dt}(\phi_{,e})x^e_{,E} + \phi_{,s}\, \dot{x}^s_{,E} , \tag{1.12}$$

Multiplying (1.12) by $X^E_{,g}$, using (0.8), and rearranging some indices gives

$$\frac{d}{dt}(\phi_{,e}) - \left(\frac{d\phi}{dt}\right)_{,e} = -\phi_{,k}\, \dot{x}^k_{,e} ,$$

$$\frac{d}{dt}(\text{grad } \phi) - \text{grad } \left(\frac{d\phi}{dt}\right) = -\text{grad } \phi \,.\, \text{grad } \dot{\underset{\sim}{x}} , \tag{1.13}$$

which is the desired result.

The equations (1.10) and (1.13) are equivalent. It will be shown that either can be used in deriving a certain special but important type of convection-diffusion theorem.

2. GENERAL CONVECTION-DIFFUSION THEOREMS

A survey of recent developments in convection-diffusion theory and a study of the properties of a convection-diffusion theorem of a very general type have been given in [1]. In this section I demonstrate that, given a property of a particle of a continuum, with the property expressed as a tensor of a certain form, there is always a convection-diffusion theorem for that property. Furthermore, this convection-diffusion theorem is a special case of the type studied in [1].

Consider a composite deformation gradient $F^{a_1 \ldots a_n}_{A_1 \ldots A_n}$ (no summation on n). An explicit form for its material derivative has been given in [1]. It is

$$\dot{F}^{a_1 \ldots a_n}_{A_1 \ldots A_n} = \sum_{i=1}^{n} \dot{x}^{a_i}{}_{,A_i} F^{a_1 \ldots a_{i-1}}_{A_1 \ldots A_{i+1}} \, {}^{a_{i+1} \ldots a_n}_{A_{i+1} \ldots A_n} . \tag{2.1}$$

An easy calculation leads to

$$\dot{F}^{a_1 \ldots a_n}_{A_1 \ldots A_n} = \sum_{i=1}^{n} \dot{x}^{a_i}{}_{,v} F^{a_1 \ldots a_{i-1}}_{A_1 \ldots A_{i-1}} \, {}^{v}_{A_i} \, {}^{a_{i+1} \ldots a_n}_{A_{i+1} \ldots A_n} ; \tag{2.2}$$

or

$$\dot{F}^{a_1 \ldots a_n}_{A_1 \ldots A_n} = \sum_{i=1}^{n-1} \dot{x}^{a_i}{}_{,v} F^{a_1 \ldots a_{i-1}}_{A_1 \ldots A_{i-1}} \, {}^{v}_{A_i} \, {}^{a_{i+1} \ldots a_n}_{A_{i+1} \ldots A_n}$$

$$+ \dot{x}^{a_n}{}_{,v} F^{a_1 \ldots a_{n-1}}_{A_1 \ldots A_{n-1}} \, {}^{v}_{A_n} . \tag{2.3}$$

Defining $S^{a_1 \ldots a_n}_{A_1 \ldots A_n}$ in the obvious manner, we have

$$\dot{F}^{a_1 \ldots a_n}_{A_1 \ldots A_n} = S^{a_1 \ldots a_n}_{A_1 \ldots A_n} + \dot{x}^{a_n}{}_{,v} F^{a_1 \ldots a_{n-1}}_{A_1 \ldots S_{n-1}} {}^{v}_{A_n} . \tag{2.4}$$

Consider a tensor with covariant components $a_{a \ldots b}$. Form the material expression $a_{a \ldots b} F^{a \ldots b}_{A \ldots B}$. Then

$$\frac{d}{dt}(a_{a \ldots b} F^{a \ldots b}_{A \ldots B}) = \dot{a}_{a \ldots b} F^{a \ldots b}_{A \ldots B} + a_{a \ldots b} \dot{F}^{a \ldots b}_{A \ldots B} . \tag{2.5}$$

Integrating along the path of a particle yields

$$a_{a \ldots b} F^{a \ldots b}_{A \ldots B} = A_{A \ldots B} + \int_0^t (\dot{a}_{a \ldots b} F^{a \ldots b}_{A \ldots B} + a_{a \ldots b} \dot{F}^{a \ldots b}_{A \ldots B}) dt, \tag{2.6}$$

where $A_{A \ldots B}$ is the value of $a_{a \ldots b}$ at $t=0$. Using the definitions (0.12) and (0.13) and the property (0.8), we obtain from (2.2) the <u>general convection-diffusion theorem</u>

$$a_{a \ldots b} = \left[A_{A \ldots B} + \int_0^t (\dot{a}_{c \ldots d} F^{c \ldots d}_{A \ldots B} + a_{c \ldots d} \dot{F}^{c \ldots d}_{A \ldots B}) dt \right] f^{A \ldots B}_{a \ldots b} . \tag{2.7*}$$

This equation states that the present value of $\underset{\sim}{a}$ associated with a particular particle is the result of two processes. The first, expressed by the term

$$A_{A \ldots B} f^{A \ldots B}_{a \ldots b}$$

is the shift of the initial value of $\underset{\sim}{a}$ to its present position, and is independent of the intervening motion. It is called <u>convection</u>.

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*This can be further simplified by using (2.2).

The second process, expressed by the integral

$$\int_0^t (a_{c\ldots d}\, \dot{F}^{c\ldots d}_{A\ldots B} + a_{c\ldots d}\, F^{c\ldots d}_{A\ldots B})dt\, f^{A\ldots B}_{a\ldots b}$$

is called <u>diffusion</u>. It is seen that diffusion is a functional of the histories of $\underset{\sim}{a}$ and the motion. It has been pointed out by Passman [4] that convection and diffusion are not unique processes.

The special case of (2.7) where $\underset{\sim}{a}$ can be written as the spatial gradient of another tensor is of considerable interest in continuum mechanics. Let there exist a tensor $\underset{\sim}{b}$ such that

$$a_{a\ldots be} = b_{a\ldots b,e} \,. \tag{2.8}$$

The appropriate form of (2.7) is

$$b_{a\ldots b,e} = \Big[B_{A\ldots B,E} + \int_0^t (\overline{\dot{b}_{c\ldots d,f}}\, F^{c\ldots df}_{A\ldots BE} + b_{c\ldots d,f}\, \dot{F}^{c\ldots df}_{A\ldots BE})dt \Big] f^{A\ldots BE}_{a\ldots be} \,. \tag{2.9}$$

By (1.13), this becomes

$$b_{a\ldots b,e} = \Big[B_{A\ldots B,E} + \int_0^t ([\dot{b}_{c\ldots d,f} - b_{c\ldots d,u}\, \dot{x}^u{}_{,f}]\, F^{c\ldots df}_{A\ldots BE} + b_{c\ldots d,f}\, \dot{F}^{c\ldots df}_{A\ldots BE})dt \Big] f^{A\ldots BE}_{a\ldots be} \,. \tag{2.10}$$

Substituting (2.4) into (2.10) gives

$$b_{a\ldots b,e} = \Big[B_{A\ldots B,E} + \int_0^t (\dot{b}_{c\ldots d,f}\, F^{c\ldots df}_{A\ldots BE} + b_{c\ldots d,f}\, S^{c\ldots df}_{A\ldots BE})dt \Big] f^{A\ldots BE}_{a\ldots be} \,. \tag{2.11}$$

Most of the convection-diffusion theorems familiar in continuum mechanics are consequences of (2.11).

The important relation (2.11) can be derived by an alternate method. Form the material expression $b_{a\ldots b,E}\, F^{a\ldots b}_{A\ldots B}$. By the commutation formula (1.10)

$$\frac{d}{dt}(b_{a\ldots b,E}\, F^{a\ldots b}_{A\ldots B}) = \dot{b}_{a\ldots b,E}\, F^{a\ldots b}_{A\ldots B} + b_{a\ldots b,E}\, \dot{F}^{a\ldots b}_{A\ldots B}\,, \tag{2.12}$$

which by the generalization of (1.4), is

$$\frac{d}{dt}(b_{a\ldots b,e}\, F^{a\ldots be}_{A\ldots BE}) = \dot{b}_{a\ldots b,e}\, F^{a\ldots be}_{A\ldots BE} + b_{a\ldots b,e}\, \dot{F}^{a\ldots b}_{A\ldots B}\, F^{e}_{E}\ . \tag{2.13}$$

The result (2.11) follows by the obvious sequence of steps.

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13. ABSTRACT

A plausibility argument is given for the existence of a certain commutation formula is given. This formula is then used to derive a general type of convection - diffusion theorem which generalizes a classical formula in kinematical vorticity theory.

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